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Threshold Adaptation in Spectrum Sensing For Cognitive Radio using Particle Swarm Optimization

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***Abstract*—Cognitive radio concerns primarily with the detection of primary (licensed) users in the radio frequency spectrum. In the case of a primary user is not transmitting in its allocated frequency band, the band is made available to a secondary (unlicensed) user for better spectrum utilization. However, the threshold of detecting a primary user should take into consideration sensing error minimization which includes minimum interference on the primary users and high spectrum utilization. In this paper, an improved energy detector equipped with particle swarm optimization is proposed to dynamically adapt the decision threshold according to the received signal conditions. The problem is formulated as a multi-dimensional constrained optimization and solved using penalty functions approach. Simulation results have shown significant improvement over the conventional energy detection technique that uses fixed threshold**

Keywords*—*Cognitive radio, energy detection, dynamic threshold, particle swarm optimization

I. INTRODUCTION

Cognitive radio (CR) is one of the latest technologies proposed to support high demands on the radio frequency spectrum [1] and [2]. CR system is supposed to enable frequency reuse when the allocated frequency bands for the (licensed), also called primary, users are not being used. On the other hand, secondary users can use these spectrum holes provided that it guarantees minimum interference on the primary users.

The performance of the CR system is measured in terms of probability of detection, P_D and the probability of false alarm, P_F. P_D measures the ability of the system to detect the presence of primary users in the spectrum, high detection rate provides maximum protection to the primary users (PU) from being affected by secondary users transmission. On the other hand, P_F measures the rate at which a primary user can be detected while it is not truly transmitting, low probability of false alarm provides high spectrum utilization. Therefore, for best performance, CR system should provide maximum detection and minimum false alarm probabilities through spectrum sensing techniques [3].

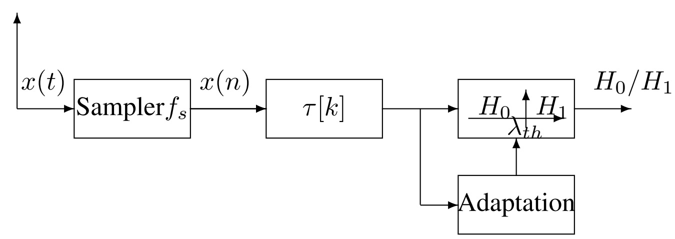

Fig. 1. An Energy Detector

There have been several spectrum sensing techniques proposed in literature. They include energy detection [1], [4] and [5], waveform and the cyclostationary based sensing [1]. Due to its simplicity and ease of implementation, energy detection (ED) has been the most common sensing technique. In ED, the energy of the observed signal is taken as the test statistics, $\tau(k)$, which can be measured in time [2] or frequency domain [4] and compared to a threshold, λ_{th}. Carriers with energy above a specified threshold are declared to be active, otherwise, if the carriers are absent, their frequency bands are made available to the secondary users. A typical energy detector is as shown in Figure 1.

It should be noted in Figure 1 that in conventional energy detector the threshold adaptation is not used and the threshold selection is relied on a specified probability of false alarm. Therefore, given the correspondence between P_D and P_F, the threshold is fixed and independent of the observed signal variations. In this case, if the SNR of the received signal is low, then sensing performance will be far from optimal. To solve the problem, adaptive threshold techniques have been proposed.

In [2], a gradient based threshold estimation has been presented. The threshold is estimated for each signal variation by minimizing P_F and maximizing P_D or equivalently minimizing the probability of a missed detection, P_M. However, in order to track the received signal variations, long sensing interval is needed in addition to high complexity that arise from the gradient calculation. These requirements are not affordable in some cases such as in battery powered devices.

In [6], the estimation of the detection threshold is performed using 1 D Particle Swarm Optimization (PSO) algorithm [8]. In that work, PSO is used in cooperative scheme to estimate a fixed global threshold for each observation using the global signal to noise ratio which is assumed to be known at the receiver. In that case, the same threshold is used in the detection of all primary users without considering received signal variations, which in turns, falsifies detection such as in the case of large primary user being transmitting on the spectrum.

In addition to that, the spectrum sensing constraints have not been considered. The authors in that article have calculated the probability of error using fixed carrier status pattern while the receiver operating characteristic (ROC) of the system has not been shown.

In this context, we consider the estimation of the adaptive threshold in the energy detection as the transmission condition changes, i.e., primary user transmission and signal to noise ratio changes. The proposed algorithm assigns a distinct (local) threshold to each frequency band of interest such that each carrier status is decided separately using its local signal to noise ratio which increases the problem dimensionality with number of primary users. Thresholds are estimated as to minimize the probability of false alarm and the probability of a missed detection while satisfying spectrum sensing constraints. Specifying spectrum constraints in the current problem is an important issue since it determines the technology where the energy detector is to be used.

In order to avoid high computational burden as the number of primary user increases, PSO is employed with penalty functions approach to deal with spectrum sensing constraints. To our knowledge, the application of PSO to this kind of problem with a single non-cooperative receiver scheme and local detection thresholds has not been addressed before. The proposed algorithm is evaluated and the ROC is determined with random carrier status using Monte-Carlo simulation.

The rest of the paper is organized as follows: in section II we present energy detection system model and formulate the problem. PSO algorithm and problem encoding by PSO are shown in section III. Simulation results are shown in section IV, and finally we conclude in section V.

II. System Model

Consider T frames of a baseband signal $x(n)$ each of N samples received at the CR receiver [4]. Where $t = 0, 1 \ldots, T-1, n = 0, 1, \ldots N-1$. The power spectral density (PSD) for the signal $x_t(n)$ is given by

$$P_t(k) = |X_t(k)|^2 \tag{1}$$

where $X_t(k)$ is the discrete time Fourier transform of the signal $x_t(n)$ calculated at frequencies $k\frac{fs}{N}$, $k = 0, 1 \ldots, N/2$ due to Fourier transform symmetry. The average PSD of the T frames is given by

$$\bar{P}(k) = \frac{1}{T} \sum_{t=0}^{T-1} P_t(k) \tag{2}$$

Instead of calculating the test statistics as the global signal energy as in [6], the test statistics, $\tau(k)$ of the CR receiver is calculated locally as the energy of each PU frequency band centered at $k\ fs/N$, i.e.,

$$\tau(k) = \frac{1}{N} \bar{P}(k) \tag{3}$$

Detecting the status of a primary user in the spectrum can be formulated as a binary hypothesis test as

$$\begin{aligned} H_1 : x_t(n) &= c\ s_t(n) + v_t(n) \\ H_0 : x_t(n) &= v_t(n) \end{aligned} \tag{4}$$

where c is the channel gain between the primary and the secondary users. $s(n)$ is the transmitted signal in each frame assumed to be (iid), zero mean, σ_s^2 variance and uncorrelated with noise samples $v(n)$. Similarly, $v(n)$ is assumed to be a zero mean σ_v^2 variance Gaussian random process, i.e., $v(n) \sim\ N(0, \sigma_v^2)$.

A primary carrier is declared to be active or idle according to the decision rule given by

$$\tau(k) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda(k) \tag{5}$$

The threshold λ is considered to be constant in the conventional energy detection, i.e. $\lambda(k) = \lambda$ and determined by a specified probability of false alarm, P_F.

The probability of detection and the probability of false alarm can be calculated for AWGN channels as in [5]

$$\begin{aligned} P_D &= \mathrm{P_r}\left(\tau(k) > \lambda(k) | H_1\right) \\ &= Q_N\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} P_F &= \mathrm{P_r}\left(\tau(k) > \lambda(k) | H_0\right) \\ &= \frac{\Gamma(N/2, \lambda/2)}{\Gamma\left(N/2\right)} \end{aligned} \tag{7}$$

where γ is the global signal to noise ratio (SNR) of the received signal. $Q_N(.\ ,\ .)$ is the generalized Marcum-Q function with N degrees of freedom, $\Gamma(.\ ,\ .)$ is the incomplete Gamma function and $\Gamma(.)$ is the complete Gamma function.

In the calculation of P_D, the signal to noise ratio is required. If the noise variance σ_v^2 is known a priori using vacant spectrum measurements, for instance or as in the case the PU user power is known e.g. IEEE 802.22 standard [7], then the signal power can be calculated by subtracting σ_v^2 from the total received power.

It should be noted from equations (6) and (7) that P_F is not a function of the signal to noise ratio. For best performance, P_F should be kept as small as possible, smaller values of P_F means higher threshold $\lambda(k)$ which will produce a lower P_D

and hence poor detection capability of weak primary users. Similarly, higher P_D means smaller thresholds and higher possibility of declaring an idle primary user as an active which produces higher P_F.

To avoid the problem, P_F and $P_M = 1 - P_D$ should be minimized for a given SNR. In conventional adaptive threshold techniques such as in [6], the global SNR is used in each observation to determine a single threshold that used to assess the status of all primary users. In this work, minimization tends to estimate a threshold vector $\Lambda(k) = [\lambda_1,\, \lambda_2,\, \ldots \lambda_k]$ such that the average error probability over all primary user bands is minimized using the local SNR of each user, γ_k. In this case, the average sensing error probability for the k^{th} user can be calculated as

$$\begin{aligned} \bar{P}_e &= \bar{P}_M + \bar{P}_F \\ &= \frac{1}{2}\left[1 - Q_N\left(\sqrt{2\gamma_k}, \sqrt{\lambda_k}\right)\right] \\ &+ \frac{1}{2}\left[\frac{\Gamma(N/2, \lambda_k/2)}{\Gamma\left(N/2\right)}\right] \end{aligned} \tag{8}$$

Interestingly, the objective function to be minimized is $\bar{P}_e$, if the SNR, γ_k, in a given band is high, then it can be shown that the threshold λ_k in this band is low for high P_D but should not be too low as this would increase P_F. Therefore, considering the conflicting requirements in optimizing (8) for a given threshold, it is necessary to impose constraints on the $\bar{P}_M$ and $\bar{P}_F$ as to control the level of optimization which is also necessary in designing a detector with specified error probabilities [2]. The optimization process can be stated as

$$\begin{aligned} &\underset{\lambda_k, \gamma_k}{\text{minimize}} && \bar{P}_e(\lambda_k, \gamma_k) \\ &\text{subject to} && \bar{P}_M(\lambda_k, \gamma_k) \leq \alpha \\ & && \bar{P}_F(\lambda_k) \leq \beta. \end{aligned} \tag{9}$$

The constrained optimization can be turned into unconstrained optimization using the penalty functions approach [9]. The augmented fitness (cost) function can be written as

$$\begin{aligned} \text{Fitness}(\lambda_k) &= \bar{P}_e + P_1 \max\left\{0, \bar{P}_M(\lambda_k) - \alpha\right\}^2 \\ &+ P_2 \max\left\{0, \bar{P}_F(\lambda_k) - \beta\right\}^2 \end{aligned} \tag{10}$$

where P_1 and P_2 are the penalty factors selected as large numbers to prevent $\bar{P}_e$ from dominating the augmented cost and therefore returns Λ that is not a feasible solution to the problem. Therefore, each constrain violation adds a high cost to the objective function proportional to the deviation from the feasible value.

The optimization in (10) is a $k - D$ problem which is the main difference between this work and the other solutions to the problem, such that each dimension represents an independent status of each primary user. The optimal values of Λ need multi-dimensional exhaustive search with random initialization in order to be determined. Particle Swarm Optimization will be used to estimate threshold vector Λ.

Algorithm 1: Adaptive threshold estimation using PSO

for *For each user* **do**
  repeat
    for $j = 1$ *to swarm size* **do**
      if *fitness*(λ_j,γ) $\leq$ *fitness*(λ_{jpbest},γ) **then**
        $\lambda_{jpbest} = \lambda_j$;
        $fitness(\lambda_{jpbest}) = fitness(\lambda_j)$;
      end
    end
    $\lambda_{gbest} = \arg\min\limits_{\lambda_j}(fitness)$;
    for $j = 1$ *to swarm size* **do**
      UpdateVelocity:

$$\begin{aligned} v_j(n+1) &= wv_j(n) \\ &+ C_1 r_1(n)\left[\lambda_{jpbest}(n) - \lambda_j(n)\right] \\ &+ C_2 r_2(n)\left[\lambda_{gbest}(n) - \lambda_j(n)\right] \end{aligned}$$

      UpdatePosition:

$$x_j(n+1) = x_j(n) + v_j(n+1)$$

    end
  until *StopCondition is met* ;
end

III. Threshold Optimization using Particle Swarm Optimization

Particle Swarm Optimization has been designed to optimize multi-dimensional stochastic functions [8]. It simulates the natural behavior of bees searching for flowers in a field. Each particle, λ_i is attracted to the position of the particle that encountered the best result globally, λ_{gbest} and affected by its own best experience in exploring the field, λ_{ipbest}. The attraction is made by a certain velocity $v(n)$ which is determined according to the quality of the particle's current result and the best particle in the swarm.

The PSO algorithm is used to estimate the threshold λ_i , $i = 1 : k$ as shown in algorithm 1.

In the algorithm, C_1 and C_2 are the social parameters selected as $C_1 + C_2 \leq 4$ to provide stability as the particle moves. w is the inertial weight decreased linearly with time to provide better exploration in the first few iterations.

IV. Simulation Results

To evaluate the performance of the proposed algorithm, we set $\bar{P}_M \leq 0.2$, $\bar{P}_F \leq 0.1$, such parameters are used in an aggressive cognitive radio system. The signal to noise ratio SNR is set to -15 dB to all active carriers. The frequency band of interest is in the range $[10\ ,\ 32]$ MHz with bandwidth 2 MHz per carrier which gives a total of 12 channels in the spectrum. The PSO parameters are set as iterations $=$ 100, $w = [0.8\ ,\ 0.4]$, swarm size $= 28$. The PSO search space is set in the range $[0,\ 1.5 \times \max\left\{\tau(k)\right\}]$.

The received signal at the CR input is generated using random carrier status in each trial and 100 Monte-Carlo trials are evaluated. The PSO is invoked in each trial to estimate

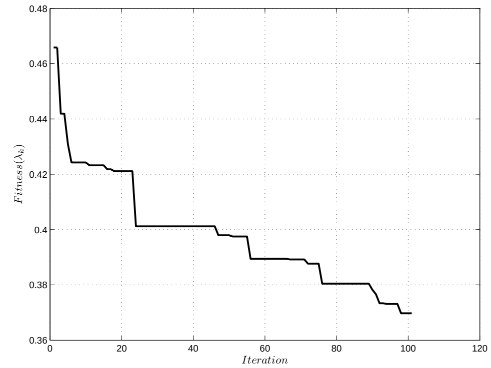

Fig. 2. Fitness $SNR = -15$ dB

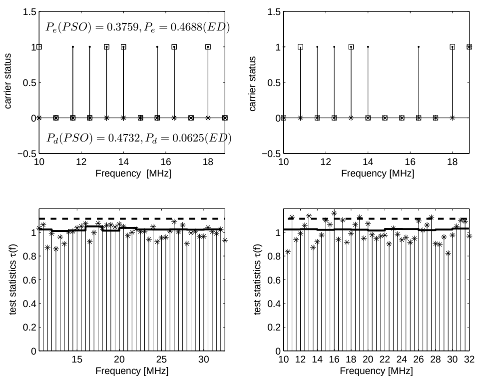

Fig. 3. Carrier status with test statistics for $SNR = -15$ dB

$[\lambda_1, \lambda_2 \dots \lambda_{12}]$ that minimizes the sensing error (9) while satisfying the imposed constraints. The fitness function is as shown in figure 2. It is obvious that fitness starts with a high value results from constraints violation and then the PSO adapts the thresholds λ_k such that the fitness is minimized while the spectrum constraints are satisfied. It should be noted here that the algorithm converges to a minimum sensing error of 0.37 for some threshold values but might not converge in all dimensions due to low SNR.

To illustrate threshold adaptation using the proposed technique, figure 3 shows two Monte-Carlo trials using random primary user status (active/idle) along with the test statistics $\tau(k)$ in each observation. In the first row, the status of the primary carriers is shown such that the original status (dotted), estimated status using the proposed technique (squared) and the estimated status using fixed threshold energy detection (asterisk). In the second row, $\tau(k)$ is shown with fixed (dashed) and adaptive (solid) thresholds. As it can be seen from the figure, adaptive threshold is at about 47% detection probability compared to almost 0 for the ED with fixed threshold.

When designing a detector, it is important to evaluate the false alarm probability as this can under utilize the spectrum. Therefore, the probability of error is calculated as (8), for example, in the first trial it was 0.376 compared to 0.468 for the ED, which confirms the performance of the proposed technique compared to the conventional ED.

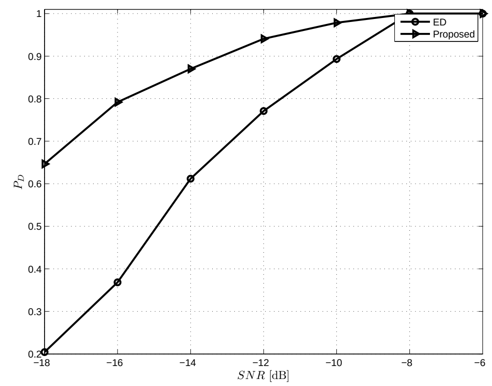

Fig. 4. Probability of detection , P_D versus SNR

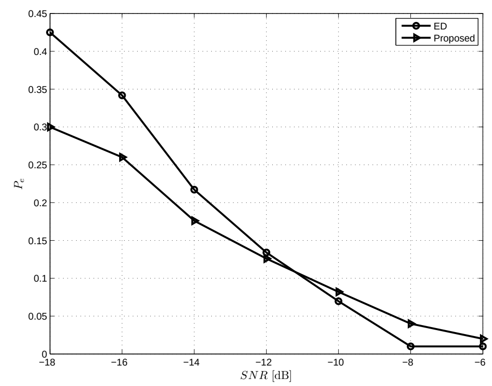

Fig. 5. Probability of error, P_e versus SNR

Finally, the performance of the proposed detector is evaluated versus signal to noise ratio SNR using the probability of detection, $P_D = P_r\left\{\hat{k} = k | k = 1\right\}$ averaged over all trials, and the total average error measures. The probability of detection is as shown in figure 4.

As it can be seen from figure 4, the proposed detector outperforms the conventional ED for small SNR (below -8 dB) and ED meets the same performance only at -8 dB. To confirm the performance increase the probability of error is evaluated for different SNR as shown in figure 5.

The total error shows performance increase as it offers good spectrum utilization while keeping very good detection rate. It should be noted that the error for the conventional ED is lower for SNR above -11 dB. However, this does not mean better performance since we have controlled the optimization level as $\bar{P}_f \leq 0.1$ which is already achieved in our proposed technique.

V. CONCLUSION

In this paper we have presented an improved energy detector to solve spectrum sensing problem. The detector is based on the sensing error minimization while satisfying imposed constraints. The use of PSO is presented to solve a constrained optimization. Performance were evaluated by the use of Monte-Carlo simulation where it has shown significant performance increase over conventional ED. The performance can be further increased using more advanced techniques to estimate the signal to noise ratio since we have used the simple periodgram which is known to be a biased estimate for the SNR.

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